AIAA 82-4263

Dynamic Response of a Rectangular Plate to a Bending Moment Distributed Along the Diagonal

Daniel J. Gorman*
University of Ottawa, Ottawa, Canada

An exact analytical solution is developed for the steady-state response of an undamped, simply supported rectangular plate subjected to a harmonic bending moment distributed along the diagonal. The solution obtained is shown to be composed of remarkably simple analytical expressions. A number of verification tests have been conducted in order to demonstrate the validity of the results. In addition to resolving an engineering problem of classical interest, it is expected that this solution will play a vital role in the free-vibration analysis of triangular, quadrilateral, and other irregularly shaped plates.

Nomenclature

a,b	= plate dimensions
D	= plate flexural rigidity $= Eh^3/[12(1-\nu^2)]$
E	= Young's modulus of plate material
h	= plate thickness
M	= amplitude of applied bending moment
M^*	= dimensionless applied bending moment amplitude $= 2M\phi^2/D$
u,v	= coordinates of applied differential bending moment
u^*	$= u/a$
v^*	$= v/b$
w	= plate lateral displacement divided by sidelength a
x,y	= plate spatial coordinates
η	$= x/a$
ξ	$= y/b$
ϕ	= plate aspect ratio $= b/a$
λ^2	$= \omega a^2\sqrt{\rho/D}$
ρ	= mass of plate per unit area
ω	= circular frequency of plate vibration
ν	= Poisson ratio of plate material

Introduction

THE problem of obtaining exact solutions for the response of rectangular plates subjected to harmonic forces or moments distributed along the diagonals was discussed by Stanisic.[1] He obtained a solution for a rectangular plate subjected to a distributed time-varying load along the diagonal by means of a Fourier-finite sine transform. In a later paper[2] the present author published a Navier-type solution for simply supported plates subjected to harmonic forces distributed along the diagonal. In a subsequent paper[3] a solution of the Lévy type was developed for the same problem. Advantages of this latter solution were discussed. Of particular interest is the fact that solutions of the Lévy type are not restricted to plates with simple support along all edges. Furthermore, they involve only single series summations and thereby minimize convergence problems.

In this paper we address the much more difficult problem of obtaining the response of a simply supported plate subjected to a harmonic bending moment distributed along the diagonal. Initially a Lévy-type solution is obtained for the plate response to a concentrated bending moment. Subsequently, using the first solution as a type of Green's function, a solution is obtained for the response of the plate to a distributed bending moment. Finally plate response to individual even and odd Fourier components in the bending moment distribution is examined.

Received Oct. 19, 1981; revision received March 19, 1982.

*Professor, Dept. of Mechanical Engineering.

Analytical Procedure

Consider the simply supported rectangular plate subjected to the concentrated harmonic bending moment as shown in Fig. 1. The moment has amplitude M and circular frequency ω and is represented in conventional vector form as shown in the figure. It is applied at the coordinates u_1, v_1.

In order to obtain the response to this concentrated moment, we divide the plate into two segments and the moment into two components as indicated in Fig. 2. Here we have introduced the dimensionless coordinates ξ and η and the dimensionless distances u and v. It is known that Lévy-type solutions for segments I and II can be written, respectively, as[4]

$$w_1(\xi,\eta) = \sum_{m=1,2}^{K^*} (A_m \sinh\beta_m\eta + B_m \sin\gamma_m\eta)\sin m\pi\xi + \sum_{m=K^*+1}^{\infty} (A_m \sinh\beta_m\eta + B_m \sinh\gamma_m)\sin m\pi\xi \tag{1}$$

and

$$w_2(\xi,\eta) = \sum_{m=1,2}^{K^*} (C_m \sinh\beta_m\eta + D_m \sin\gamma_m\eta)\sin m\pi\xi + \sum_{m=K^*+1}^{\infty} (C_m \sinh\beta_m\eta + D_m \sinh\gamma_m\eta)\sin m\pi\xi \tag{2}$$

where

$$\beta_m = \phi\sqrt{\lambda^2 + (m\pi)^2}$$

$$\gamma_m = \phi\sqrt{\lambda^2 - (m\pi)^2} \quad \text{or} \quad \phi\sqrt{(m\pi)^2 - \lambda^2}$$

whichever is real, and the first summations contain only those terms for which $\lambda^2 > (m\pi)^2$.

Each term in Eqs. (1) and (2) contains two constants to be determined. Two of these constants are obtained through enforcement of the conditions of continuity of displacement and slope across the interface of the segments. The remaining two constants are evaluated by enforcing the conditions of bending-moment and twisting-moment equilibrium as they relate to the same interface. We focus our attention now on these latter two conditions.

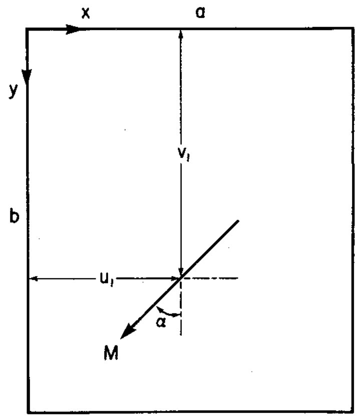

Fig. 1 Simply supported rectangular plate subjected to a concentrated harmonic bending moment of amplitude M at coordinates u_1, v_1.

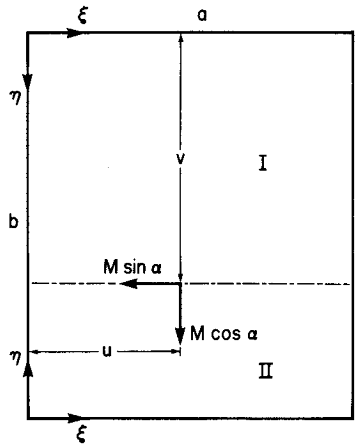

Fig. 2 Division of the plate into two segments with the common boundary passing through the point of application of the moment.

Beginning with the bending-moment component $M \sin\alpha$, it follows that we may write, in the conventional coordinates of Fig. 1,

$$\frac{-M \sin\alpha}{D} = \frac{\partial^2 w_1(x,y)}{\partial y^2} - \frac{\partial^2 w_2(x,y)}{\partial y^2} \tag{3}$$

or, in dimensionless coordinates,

$$\frac{\partial^2 w_1(\xi,\eta)}{\partial \eta^2} - \frac{\partial^2 w_2(\xi,\eta)}{\partial \eta^2} = -\frac{Mb^2}{aD}\sin\alpha \tag{4}$$

Here the bending and twisting moments may be considered as distributed along the interface but highly concentrated at the point of application. This is accomplished by means of the Dirac function, as discussed below.

Turning next to the twisting-moment component $M \cos\alpha$ and guided by the discussion of plate boundary conditions as presented by Timoshenko and Woinowsky-Krieger,[5] we write, in conventional coordinates,

$$V_1(x) + V_2(x) = \frac{\mathrm{d}M(x)}{\mathrm{d}x}\cos\alpha \tag{5}$$

where $V_1(x)$ and $V_2(x)$ are the vertical edge reactions along the respective plate segments. Substituting the appropriate derivatives for those edge reactions, we may write

$$\frac{\partial^3 w_1(\xi,\eta)}{\partial \eta^3} + \frac{\partial^3 w_2(\xi,\eta)}{\partial \eta^3} = \frac{-b^3}{aD}\frac{\mathrm{d}M(x)}{\mathrm{d}x}\cos\alpha \tag{6}$$

In order to employ Eqs. (4) and (6), we follow the practice of representing the moments and their derivatives as required by means of Dirac functions. Returning to Eq. (3), we may write

$$\frac{\partial^2 w_1(x,y)}{\partial y^2} - \frac{\partial^2 w_2(x,y)}{\partial y^2} = \frac{-M\sin\alpha}{D}\frac{2}{a}\sum_{m=1}^{\infty}\sin m\pi u \sin\frac{m\pi x}{a} \tag{7}$$

or, in dimensionless form,

$$\frac{\partial^2 w_1(\xi,\eta)}{\partial \eta^2} - \frac{\partial^2 w_2(\xi,\eta)}{\partial \eta^2} = -M^*\sin\alpha\sum_{m=1}^{\infty}\sin m\pi u \sin m\pi\xi \tag{8}$$

where

$$M^* = 2M\phi^2/D$$

Turning to Eqs. (5) and (6), it is seen that we require a Dirac-type representation for the quantity $\mathrm{d}M(x)/\mathrm{d}x$. Furthermore, we require that this representation take the form of a sine series. To achieve this end, we begin by expanding the moment $M(x)$ in a cosine series as

$$M(x) = M\cos\alpha\left(\frac{1}{a} + \frac{2}{a}\sum_{m=1}^{\infty}\cos m\pi u \cos\frac{m\pi x}{a}\right) \tag{9}$$

Taking the appropriate derivatives, we may rewrite Eq. (5) as

$$\frac{\partial^3 w_1(x,y)}{\partial y^3} + \frac{\partial^3 w_2(x,y)}{\partial y^3} = \frac{2M\cos\alpha}{aD}\sum_{m=1}^{\infty}\frac{m\pi}{a}\cos m\pi u \sin\frac{m\pi x}{a} \tag{10}$$

or, in dimensionless form,

$$\frac{\partial^3 w_1(\xi,\eta)}{\partial \eta^3} + \frac{\partial^3 w_2(\xi,\eta)}{\partial \eta^3} = M^*\phi\cos\alpha\sum_{m=1}^{\infty}m\pi\cos m\pi u \sin m\pi\xi \tag{11}$$

Equations (8) and (11) constitute formulation of the dimensionless interface continuity conditions as they relate to bending and twisting moments, respectively. Dimensional equations leading up to these have been provided for the convenience of the reader.

Solutions for the four constants of Eqs. (1) and (2) are obtained by means of the four boundary conditions described previously. Two sets of equations are required, one for the condition $\lambda^2 > (m\pi)^2$ and one for the condition $\lambda^2 < (m\pi)^2$. In view of the fact that orderly procedures for obtaining solutions to these sets of equations have already been described by the author,[4] only the results are provided here.

For the condition $\lambda^2 > (m\pi)^2$

$$A_m = -M^*\frac{\{\beta_m\sin\alpha\sin m\pi u\cosh\beta_m v^* - \phi m\pi\cos\alpha\cos m\pi u\sinh\beta_m v^*\}}{(\beta_m^2+\gamma_m^2)\beta_m\sinh\beta_m}$$

$$B_m = -M^*\frac{\{\phi m\pi\cos\alpha\cos m\pi u\sin\gamma_m v^* - \gamma_m\sin\alpha\sin m\pi u\cos\gamma_m v^*\}}{(\beta_m^2+\gamma_m^2)\gamma_m\sin\gamma_m}$$

$$C_m = \frac{M^*\{\phi m\pi\cos\alpha\cos m\pi u\sinh\beta_m v + \beta_m\sin\alpha\sin m\pi u\cosh\beta_m v\}}{(\beta_m^2+\gamma_m^2)\beta_m\sinh\beta_m}$$

$$D_m = \frac{-M^*\{\phi m\pi\cos\alpha\cos m\pi u\sin\gamma_m v - \gamma_m\sin\alpha\sin m\pi u\cos\gamma_m v\}}{(\beta_m^2+\gamma_m^2)\gamma_m\sin\gamma_m} \tag{12}$$

and for the condition $\lambda^2 < (m\pi)^2$

$$A_m = -M^* \frac{\{\beta_m \sin\alpha \sin m\pi u \cosh\beta_m v^* - \phi m\pi \cos\alpha \cos m\pi u \sinh\beta_m v^*\}}{(\beta_m^2 - \gamma_m^2)\beta_m \sinh\beta_m}$$

$$B_m = -M^* \frac{\{\phi m\pi \cos\alpha \cos m\pi u \sinh\gamma_m v^* - \gamma_m \sin\alpha \sin m\pi u \cosh\gamma_m v^*\}}{(\beta_m^2 - \gamma_m^2)\gamma_m \sinh\gamma_m}$$

$$C_m = \frac{M^*\{\phi m\pi \cos\alpha \cos m\pi u \sinh\beta_m v + \beta_m \sin\alpha \sin m\pi u \cosh\beta_m v\}}{(\beta_m^2 - \gamma_m^2)\beta_m \sinh\beta_m}$$

$$D_m = \frac{-M^*\{\phi m\pi \cos\alpha \cos m\pi u \sinh\gamma_m v + \gamma_m \sin\alpha \sin m\pi u \cosh\gamma_m v\}}{(\beta_m^2 - \gamma_m^2)\gamma_m \sinh\gamma_m} \tag{13}$$

It is worthwhile noting that the constants C_m and D_m of the preceding sets can be directly inferred from the constants A_m and B_m, respectively, of the same set. In view of the symmetry involved, one need only replace the coordinate v^* with v and the angle α with $-\alpha$. This is easily verified through physical reasoning similar to that discussed in Ref. 4.

Equations (1) and (2) along with (12) and (13) allow us to establish the dynamic response of a simply supported rectangular plate to an externally applied concentrated harmonic bending moment. The bending moment may be applied anywhere on the plate lateral surface, and it may have any orientation desired.

We now turn our attention to the response of the same plate to a moment distributed along the diagonal as shown schematically in Fig. 3. Along the diagonal the parameter u equals v and the angle α is negative. Its magnitude equals that of an angle whose tangent is b/a, the plate aspect ratio. The distribution of the dimensionless bending moment M^* is represented by a sine series as

$$M^*(v) = \sum_{n=1,2}^{\infty} E_n \sin n\pi v \tag{14}$$

The moment distribution in Fig. 3 is composed of the first term of a sine series only, although any number of terms may be used as required. To illustrate the solution procedure, we

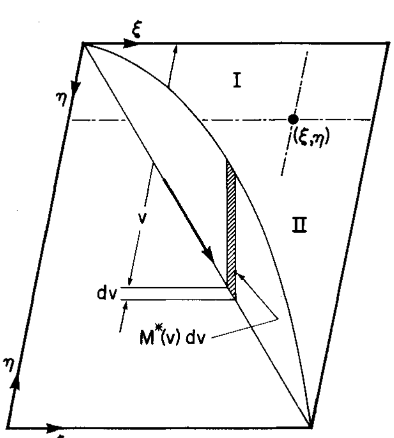

Fig. 3 Schematic representation of a rectangular plate subjected to a harmonic bending moment distributed along the diagonal. Moment as shown has only first component of Fourier sine series in its distribution.

focus our attention on the point (ξ,η) as shown in the figure with the plate divided into sections I and II.

The displacement at location (ξ,η) will equal that resulting from moment distributed in region II as well as moment distributed in region I. These are treated separately. We begin by considering the displacement resulting from moment distributed in region II. The differential displacement resulting from the differential applied moment $M^*(v)\mathrm{d}v$ as shown in Fig. 3 is given by

$$\mathrm{d}W(\xi,\eta) = \sum_{m=1,2}^{K^*} M^*(v)\mathrm{d}v[A_m(v)\sinh\beta_m\eta + B_m(v)\sin\gamma_m\eta] \times \sin m\pi\xi + \sum_{m=K^*+1}^{\infty} M^*(v)\mathrm{d}v[A_m(v)\sinh\beta_m\eta + B_m(v)\sinh\gamma_m\eta]\sin m\pi\xi \tag{15}$$

where the functions $A_m(v)$ and $B_m(v)$ are available from Eqs. (12) and (13). We recall that the quantity u appearing in these functions is now replaced by v. Collecting the contributions of all the moment differential elements of region II toward the preceding displacement, we have through integration

$$W(\xi,\eta) = \sum_{m=1,2}^{K^*} \left\{\left[\int_\eta^1 M^*(v)A_m(v)\mathrm{d}v\right]\sinh\beta_m\eta + \left[\int_\eta^1 M^*(v)B_m(v)\mathrm{d}v\right]\sin\gamma_m\eta\right\}\sin m\pi\xi + \sum_{m=K^*+1}^{\infty}\left\{\left[\int_\eta^1 M^*(v)A_m(v)\mathrm{d}v\right]\sinh\beta_m\eta + \left[\int_\eta^1 M^*(v)B_m(v)\mathrm{d}v\right]\sinh\gamma_m\eta\right\}\sin m\pi\xi \tag{16}$$

Following identical steps, it is shown that the contribution toward displacement at the point (ξ,η) of that portion of the bending moment distributed in region I is given by

$$W(\xi,\eta) = \sum_{m=1,2}^{K^*} \left\{\left[\int_0^\eta M^*(v)C_m(v)\mathrm{d}v\right]\sinh\beta_m(1-\eta) + \left[\int_0^\eta M^*(v)D_m(v)\mathrm{d}v\right]\sin\gamma_m(1-\eta)\right\}\sin m\pi\xi + \sum_{m=K^*+1}^{\infty}\left\{\left[\int_0^\eta M^*(v)C_m(v)\mathrm{d}v\right]\sinh\beta_m(1-\eta) + \left[\int_0^\eta M^*(v)D_m(v)\mathrm{d}v\right]\sinh\gamma_m(1-\eta)\right\}\sin m\pi\xi \tag{17}$$

It is evident by now that determination of the plate response to any bending moment distributed along the diagonal necessitates evaluation of the integrals of Eqs. (16) and (17). Fortunately this task is not as formidable as it might at first appear, as a number of common terms can be grouped together and others cancel each other. It would not be expedient to reproduce all the steps in the integration here. Rather, we will provide only the final expressions required to describe the plate response to a single excitation term $E_n \sin(n\pi v)$:

$$W(\xi,\eta) = E_N \sum_{m=1,2,3}^{K^*} [G_{mn_1} \sin(m+n)\pi\eta$$

$$-H_{mn_1} \sin(m-n)\pi\eta] \sin m\pi\xi$$

$$+E_N \sum_{m=K^*+1}^{\infty} [G_{mn_2} \sin(m+n)\pi\eta$$

$$-H_{mn_2} \sin(m-n)\pi\eta] \sin m\pi\xi \tag{18}$$

where

$$G_{mn_1} = \frac{-[(m+n)\pi\sin\alpha - m\pi\phi\cos\alpha]}{2(\beta_m^2+\gamma_m^2)} \times \left[\frac{1}{\beta_m^2+(m+n)^2\pi^2} + \frac{1}{\gamma_m^2-(m+n)^2\pi^2}\right]$$

$$H_{mn_1} = \frac{-[(m-n)\pi\sin\alpha - m\pi\alpha\cos\alpha]}{2(\beta_m^2+\gamma_m^2)} \times \left[\frac{1}{\beta_m^2+(m-n)^2\pi^2} + \frac{1}{\gamma_m^2-(m-n)^2\pi^2}\right]$$

$$G_{mn_2} = \frac{-[(m+n)\pi\sin\alpha - m\pi\phi\cos\alpha]}{2(\beta_m^2-\gamma_m^2)} \times \left[\frac{1}{\beta_m^2+(m+n)^2\pi^2} - \frac{1}{\gamma_m^2+(m+n)^2\pi^2}\right]$$

$$H_{mn_2} = \frac{-[(m-n)\pi\sin\alpha - m\pi\phi\cos\alpha]}{2(\beta_m^2-\gamma_m^2)} \times \left[\frac{1}{\beta_m^2+(m-n)^2\pi^2} - \frac{1}{\gamma_m^2+(m-n)^2\pi^2}\right]$$

The first summation above is composed of those terms for which $\lambda^2 > (m\pi)^2$.

It will be appreciated that Eq. (18) can be used to obtain the plate response regardless of the number of terms required in Eq. (14) to describe the applied moment amplitude. One simply sums up the contributions of each term as given by Eq. (18).

Presentation of Theoretical Results

In order to verify the solution for the plate response, a number of test studies were performed. These are described here, beginning with the simplest.

It will be apparent that, even with the moment distributed along the diagonal, it is not necessary to have the direction of the moment vector coincident with the diagonal as shown in Fig. 3. Consider the following two cases involving a square plate where the moment amplitude is given by the first term of a Fourier sine series: 1) The angle α is set equal to $\pi/2$. 2) The angle α is set equal to 0.

In case 1 only a bending moment acts at the segment interface as considered in the analysis. In case 2 only a torsion moment acts, the amplitude of the moments being equal. Physical reasoning tells us that, denoting displacements for cases 1 and 2 by subscripts *1* and *2*, respectively, we must obtain

$$W_1(\xi,\eta) = W_2(\eta,\xi) \tag{19}$$

Studies have verified that, in fact, Eq. (19) is satisfied under the conditions described. This test is also valuable in that it demonstrates that convergence occurs, apparently equally well, for either type of moment. We recall that for the torsion moment we employed the spatial derivative of a Dirac-type function. This has evidently not created any convergence problems.

Returning to the situation where the applied moment vector lies along the diagonal as indicated in Fig. 3, it will be apparent that a number of verification tests can be performed for nonsquare plates. In particular, we are interested in pairs of plates where one plate has an aspect ratio equal to the inverse of the other. Before discussing these tests, it is appropriate to examine the mode shapes of square plates driven by moments with individual Fourier components in their amplitude distribution.

In Fig. 4 the response shape is given for a square plate with a single half sine wave in the excitation moment distribution. The expected symmetry is observed. In Fig. 5, the same plate is driven by a moment with two half sine waves in its distribution along the diagonal. As would be expected, the displacements have an antisymmetric distribution with respect to each diagonal. In each case, a value of 12 has been used for the parameter λ^2 so that terms of each summation in the solution would be required.

Results of a much more severe test of the analytical solution are contained in Figs. 6 and 7. In Fig. 6, the response shape is given for a plate of aspect ratio 1.2 with a single half wave in the excitation moment distribution. Corresponding results are presented in Fig. 7 for a plate of aspect ratio 1/1.2. It will be appreciated that, with proper choice of parameters, the displacement for the plate of Fig. 7 must equal that of Fig. 6 with the variables ξ and η interchanged. Figure 6 was generated with a value of $\lambda^2 = 12.0$. Recalling that λ^2 is made nondimensional through multiplication by a^2, it follows that it must be set equal to $12/\phi^2$, i.e., 12×1.2^2, when generating Fig. 7. In this way the circular frequency ω is held constant for both plates. Two further observations must be made. In order to keep the magnitude per unit length of the applied moment M constant, the value of M^* must be adjusted. Since a value of unity was used to generate Fig. 6, a value of $1/\phi^4$ must be used in generating Fig. 7. Finally, recalling that displacement as computed is divided by side length a, the displacement computed for Fig. 7 must be multiplied by 1.2 before it is

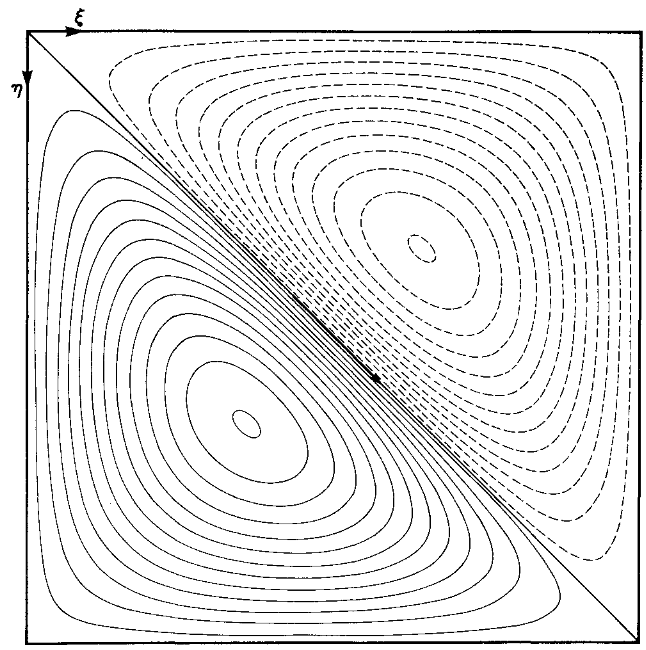

Fig. 4 Response of a square, simply supported plate to a moment distributed along the diagonal. Moment amplitude consists of the first term only of a Fourier sine series.

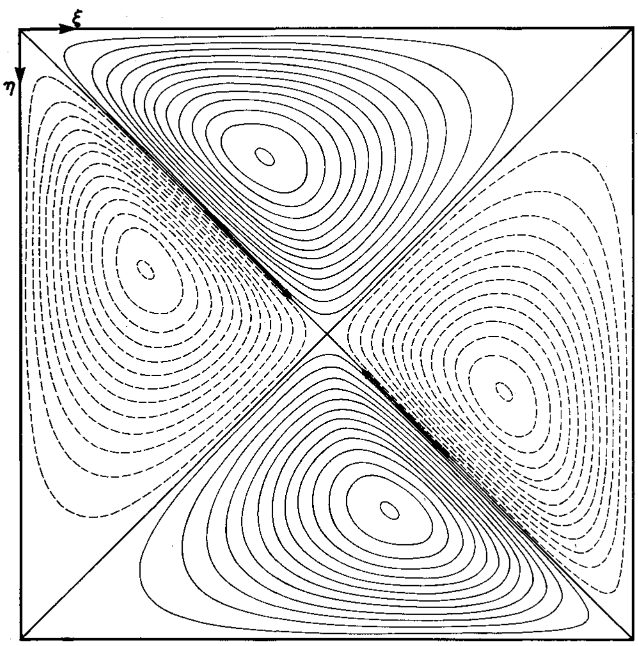

Fig. 5 Response of a square, simply supported plate to a moment distributed along the diagonal. Moment amplitude consists of the second term only of a Fourier sine series.

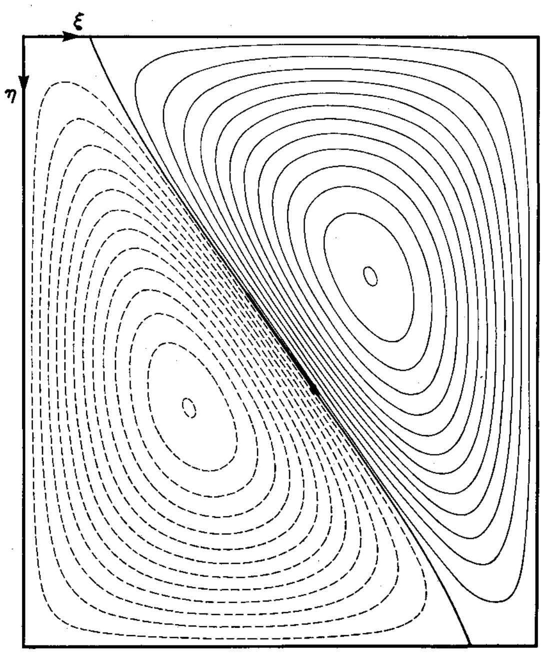

Fig. 6 Response of a simply supported plate of aspect ratio 1.2 to a moment distributed along the diagonal. Moment amplitude consists of the first term only of a Fourier sine series.

compared with that of Fig. 6. In fact, both of these latter two considerations are taken care of by simply setting M^* equal to unity and dividing the displacement computed for Fig. 7 by 1.2^3. A study of Figs. 6 and 7 indicates that the abovementioned symmetry-related conditions are completely satisfied. This has also been verified through a point-by-point study of the digital computer output. Identical verification tests have been conducted with two half waves in the excitation moment distribution. Again, fulfillment of these conditions of symmetry as previously described was verified.

As a final verification check, we examine bending-moment distributions in a plate of aspect ratio 2.0. The applied moment M^* consists of the first term of a Fourier sine series of amplitude unity distributed along the diagonal. It is of

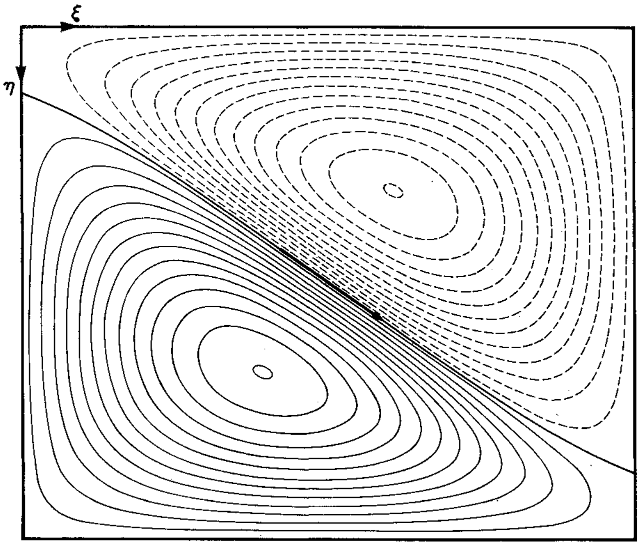

Fig. 7 Response of a simply supported plate of aspect ratio 1/1.2 to a moment distributed along the diagonal. Moment amplitude consists of the first term only of a Fourier sine series.

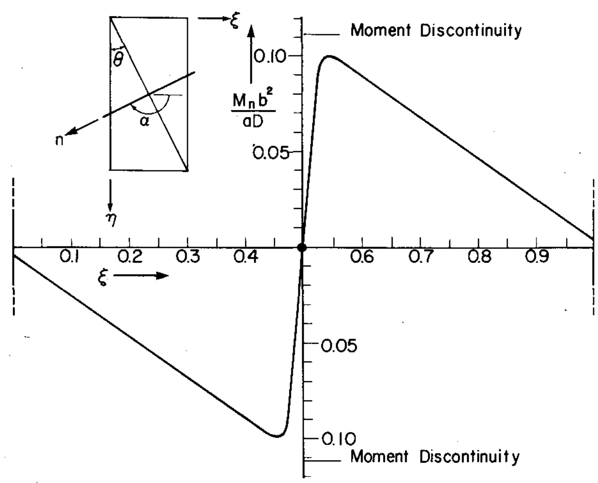

Fig. 8 Distribution of bending moment M_n along line indicated, for a rectangular plate of aspect ratio 2. Plate is subjected along the diagonal to an applied bending moment given by the first term of a Fourier sine series. Moment distribution intercepts diagonal at midway point.

interest to examine the distribution of plate bending moment M_n as shown in the inserts of Figs. 8 and 9, where both the distribution lines and the direction n run perpendicular to the diagonal.

It is easily shown that we may write

$$\frac{M_n b^2}{aD} = -\left[G_{11}\phi^2 \frac{\partial^2 w(\xi,\eta)}{\partial \xi^2} + G_{22}\frac{\partial^2 w(\xi,\eta)}{\partial \eta^2} + G_{12}\phi \frac{\partial^2 w(\xi,\eta)}{\partial \xi \partial \eta}\right] \tag{20}$$

where

$$G_{11} = \cos^2\alpha + \nu \sin^2\alpha$$

$$G_{22} = \sin^2\alpha + \nu\cos^2\alpha$$

$$G_{12} = (1-\nu)\sin 2\alpha$$

and the angle α is indicated in the figures.

A plot of bending moment M_n vs coordinate ξ is shown in Fig. 8, where the distribution line passes through the center of the plate. It is known that there exists a discontinuity in this distribution at the midway point. The series solution does not

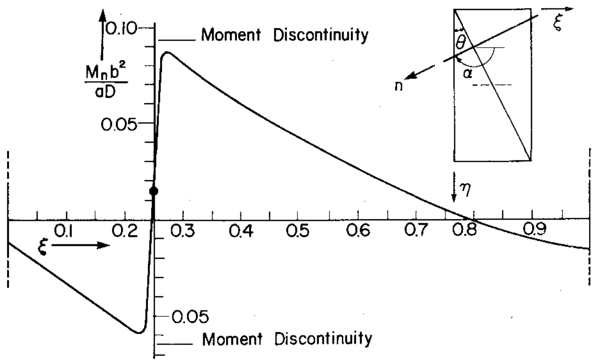

Fig. 9 Distribution of bending moment M_n along line indicated, for a rectangular plate of aspect ratio 2. Plate is subjected along the diagonal to an applied bending moment given by the first term of a Fourier sine series. Moment distribution intercepts diagonal at point one quarter way from end.

permit exact representation of the discontinuity. At the crossing of the diagonal, the series summation will take on the mean value of the function. This mean value equals zero at the center of the plate.

Returning to the definition of M^*, we now explore the value of the applied moment per unit length of diagonal. Corresponding to a differential length $d\eta$, the actual diagonal length $= b d\eta/\cos\theta$ where the angle θ is indicated in Fig. 8. Recalling that M^* equals unity at the center of the plate and rearranging the expression for M^*, we obtain

$$M_A b^2/aD = \cos\theta/2\phi \tag{21}$$

where M_A equals the applied moment per unit length. Substituting for θ and ϕ we obtain $M_A b^2/aD = 0.224$. This is the value of the discontinuity the series is attempting to approach in Fig. 8. The actual dimensionless bending moment M_n immediately to the right of the diagonal is equal to 0.112.

In Fig. 9, the distributed bending moment intercepts the diagonal at the coordinates $\xi = \eta = 0.25$. The value of M^* is, therefore, equal to $\sin(\pi/4)$. The discontinuity in the applied bending moment equals $M_A b^2/aD = 0.158$. Again the series tries to approach this discontinuity in the figure. It takes on the mean value of the bending moment at the crossing of the diagonal, which is seen to equal 0.0144.

Discussion and Conclusions

The analytical solution obtained for the response of a simply supported rectangular plate subjected to a harmonic bending moment distributed along the diagonal takes on a form that is remarkable for its simplicity. It is observed that, even if the applied moment vectors lay at some fixed angle to the diagonal, the solution would still be applicable. It will also be appreciated that the solution technique could be utilized if the applied moment were distributed along a straight line other than the diagonal. It is unlikely that the solution would take on as simple a form in this latter case, as the quantity u could not be replaced by v.

In addition to providing an exact solution for forced-vibration problems of this type, it is expected that the results of this study will prove useful in conducting free-vibration analysis of nonrectangular plates. It has already been proposed that the free vibration of right-triangular plates could be analyzed by superimposing rectangular plate solutions with lateral forces and bending moments distributed along a diagonal.[2,3] These forces and moments could be so constrained as to eliminate net displacement and bending moment along the hypotenuse of one of the right triangular segments making up the rectangular plate. Such a study is now possible since both the required rectangular plate solutions are available.

References

[1] Stanisic, M. M., "Dynamic Response of a Diagonal Line-Loaded Rectangular Plate," *AIAA Journal*, Vol. 15, Dec. 1977, pp. 1804-1807.

[2] Gorman, D. J., "Free Vibration Analysis of Rectangular Plates with Inelastic Lateral Support on the Diagonals," *Journal of the Acoustical Society of America*, Vol. 64, Sept. 1978, pp. 823-826.

[3] Gorman, D. J., "Solutions of the Lévy Type for the Free Vibration Analysis of Diagonally Supported Rectangular Plates," *Journal of Sound and Vibration*, Vol. 66, No. 2, 1979, pp. 239-246.

[4] Gorman, D. J., *Free Vibration Analysis of Rectangular Plates*, Elsevier North-Holland, New York, 1981.

[5] Timoshenko, S. and Woinowsky-Krieger, S., *Theory of Plates and Shells*, 2nd ed., McGraw-Hill, New York, 1959.